Multiple-Access Channels with Distributed Channel State Information

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***Abstract*— This paper considers a multiple-access channel with two state components, where the first sender knows only the first state component, the second sender knows only the second component, and the receiver knows both components. The notion of adaptive-rate capacity is introduced to characterize the set of achievable rates when the state is fixed in each transmission block but can vary between blocks. Single-letter characterizations of the adaptive-rate capacity region for the discrete-memoryless and the Gaussian models are established. The expected adaptive sum-rate capacity is compared to the ergodic sum-rate capacities when the senders have complete and partial state information, and to the sum-rate capacity of the compound MAC. This comparison shows that the adaptive sum-rate capacity can be close to the ergodic capacity and much higher than the sum-rate capacity of the compound MAC.**

I. Introduction

The multiple-access channel (MAC) with state has been the subject of much study in recent years due to its relevance in wireless communication systems. For example, in [1], [2], and the references therein, the Gaussian MAC with complete knowledge of the channel state at the senders and the receiver is studied. The sum-rate capacity was characterized in [1] and the capacity region was later established in [2]. Complete knowledge of the state at the senders, however, is an unrealistic assumption in wireless communications as it requires either communication between the senders or feedback control from the access point. In time-division-duplexed (TDD) wireless communication systems, such as IEEE 802.11 wireless LAN, each sender estimates its channel state from a training signal that is broadcast from an access point to all the senders. Complete knowledge of the state at the receiver, on the other hand, is quite realistic since the access point can estimate the state of all senders from a training signal. This practical consideration has motivated the investigation of a MAC where each sender knows its own state while the receiver knows all the senders' states. In [3], [4], and [5] this setting was investigated with the goal of achieving the sum-rate capacity of the Gaussian fading MAC via random access. In [6] and [7], the discrete-memoryless MAC (DM-MAC) with partial knowledge of the state at the senders and the receiver is studied. In [6], it is assumed that two compressed descriptions of a non-causal channel state are available at the senders, and the capacity region is established when one description is a function of the other. This result is generalized in [7] by considering both the causal and non-causal CSI in a unified manner.

In this paper we consider a 2-sender DM-MAC with two state components, where the first sender knows only the first state component, the second sender knows only the second component, and the receiver knows both state components. We further assume an i.i.d state model, where the state is fixed throughout each transmission block, but can vary between different blocks. This setting will be referred to as *MAC with distributed state information* (MAC-DSI). We define the *adaptive-rate capacity region* to be the set of achievable rates when each sender's rate is allowed to adapt to its own channel state. Unlike ergodic capacity, where the probability of error averaged over all state realizations is required to approach zero with increasing block length, here we require that the probability of error approaches zero for *every* state realization. We establish the adaptive-rate capacity region for both the discrete-memoryless and the Gaussian channel models. The expected adaptive sum-rate capacity is then compared to the corresponding ergodic sum-rate capacity when the state information is completely known to the senders and the receiver [1], and when only distributed state information is known at the senders (which is a special case of Theorem 5 in [7]). Additionally, the adaptive sum-rate capacity is compared to the sum-rate capacity of the compound MAC, which is the highest rate supportable in every channel state (e.g., see [8]). We show that the adaptive sum-rate capacity can be close to the ergodic sum-rate capacity and significantly better than the sum-rate under the compound channel assumption.

II. Problem Formulation

A 2-sender DM-MAC with distributed state information consists of a finite set of state pairs $\mathcal{S}_1 \times \mathcal{S}_2$ with a joint pmf $p(s_1, s_2)$ on them, and a set of DM-MACs $(\mathcal{X}_1, \mathcal{X}_2, p(y|x_1, x_2, s_1, s_2), \mathcal{Y})$, one for each state pair $(s_1, s_2) \in \mathcal{S}_1 \times \mathcal{S}_2$[1]. Without loss of generality, we assume that the two state components have the same cardinality S.

Similar to the block fading channel model defined in [9], we assume a state model where the state (S_1, S_2) is fixed during each transmission block and changes in an i.i.d. manner

[1]The extension of the definitions and results to the K-sender case is in principle straightforward.

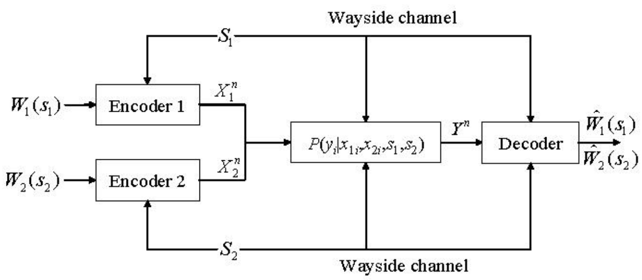

Fig. 1. Discrete memoryless MAC with distributed state information.

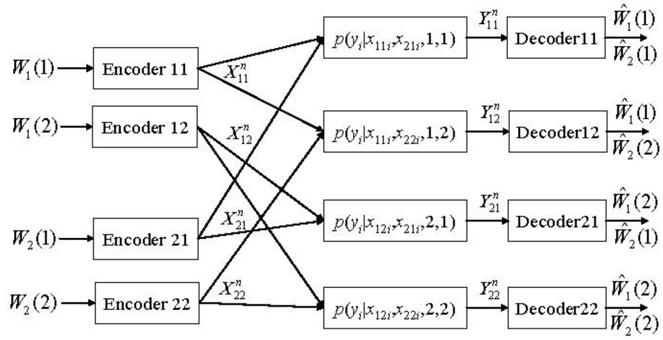

Fig. 2. DM-MAC-DSI with two states per sender.

according to the state pmf $p(s_1, s_2)$ between different blocks. The channel is memoryless; thus, without feedback the channel can be represented as

$$p(y^n|x_1^n, x_2^n, s_1, s_2) = \prod_{t=1}^{n} p\left(y_t|x_{1t}, x_{2t}, s_1, s_2\right). \qquad (1)$$

We assume that the receiver knows both state components s_1 and s_2 but that Sender 1 knows only s_1 and Sender 2 knows only s_2 before transmission commences (see Fig. 1).

This problem can be thought of as a compound MAC with distributed state information. However, the compound approach, where the codebook (but not the rate) is allowed to depend on the state can lead to a very low throughput (see Section III-A). Here, we allow both the rate and the codebook to adapt to each sender's state component to achieve high throughput for each state pair (s_1, s_2). Specifically, we define a $(2^{nR_1(1)}, \cdots, 2^{nR_1(S)}, 2^{nR_2(1)}, \cdots, 2^{nR_2(S)}, n)$ code for the DM-MAC-DSI to consist of the following components:

1) $2S$ message sets $\mathcal{W}_1(s_1) = [1, 2^{nR_1(s_1)}]$ and $\mathcal{W}_2(s_2) = [1, 2^{nR_2(s_2)}]$ for $(s_1, s_2) \in \mathcal{S}_1 \times \mathcal{S}_2$, where each pair of messages $W(s_1), W(s_2)$ is uniformly distributed over $[1, 2^{nR_1(s_1)}] \times [1, 2^{nR_1(s_2)}]$;
2) two encoders, each encoder maps each message $w_k \in \mathcal{W}_k(s_k)$ into a codeword $x_k^n(w_k, s_k)$; and
3) a decoder that maps each received sequence y^n for each (s_1, s_2) pair into an estimate $(\hat{w}_1, \hat{w}_2)$.

The average probability of error for each state pair (s_1, s_2) is defined as

$$P_e^{(n)}(s_1, s_2) = \sum_{w_1(s_1), w_2(s_2)} 2^{-n(R_1(s_1)+R_2(s_2))} \qquad (2)$$
$$P\left\{(\hat{W}_1, \hat{W}_2) \neq (w_1, w_2)|w_1, w_2, s_1, s_2\right\}.$$

The $2S$ rate tuple $(R_1(1), \cdots, R_1(S), R_2(1), \cdots, R_2(S))$ is said to be achievable if there exists a sequence of $(2^{nR_1(1)}, \cdots, 2^{nR_1(S)}, 2^{nR_2(1)}, \cdots, 2^{nR_2(S)}, n)$ codes with $P_e^{(n)}(s_1, s_2) \to 0$ for all $(s_1, s_2) \in \mathcal{S}_1 \times \mathcal{S}_2$. This guarantees reliable transmission for each state pair without being overly pessimistic as in the compound channel approach. We define the *adaptive-rate capacity region* of the DM-MAC-DSI to be the closure of the set of all achievable $2S$ rate tuples.

Note that this problem does not in general break up into a set of independent MAC capacity problems for each state pair because each sender knows only one state component and thus the rate it uses must be achievable for *all* possible state components (and the rate for each state component) of the other user. To further explain this key point, we note that the capacity region for the above setting is the same as that of the multi-MAC exemplified for $S = 2$ in Fig. 2. As shown in the figure, the multi-MAC has four senders and four receivers coupled into four MACs (each corresponding to a state component pair). Each sender has an independent message and each receiver decodes the message pair destined to it. Clearly, the capacity region of this channel, which is the closure of the set of all achievable rate quadruples, is the same as that of the MAC with distributed state information considered here. Note, however, that in our problem only one of the four MACs is present during a transmission block. Although the receiver knows which MAC is present, each sender needs to consider multiple MACs as it has only partial knowledge of the state.

In addition, we consider the AWGN MAC-DSI, where the received signal at time t is given by

$$Y_t = \sum_{k=1}^{2} H_{k,t} X_{k,t} + Z_t. \qquad (3)$$

Here $X_{k,t}$ is the transmitted signal from sender k, $H_{k,t}$ is the fading state of sender k, and Z_t is white Gaussian noise independent of the channel state. We assume a long-term average transmitted power constraint P, thus we allow power allocation over many transmission blocks as long as the average power over all blocks is no more than P.

III. DM-MAC with Distributed State Information

In this section we establish the capacity region of the DM-MAC-DSI and provide an example to compare the expected sum-rate capacity to its ergodic counterpart and to the sum-rate capacity of the compound MAC.

Theorem 1: The capacity region of the DM-MAC-DSI defined in Section 2 is the set of all rate $2S$-tuples $(R_1(1), \cdots, R_1(S), R_2(1), \cdots, R_2(S))$ that satisfy the inequalities

$$R_1(s_1) \le \min_{s_2' \in \mathcal{S}_2} I(X_1; Y|X_2, Q, s_1, s_2') \quad (4)$$

$$R_2(s_2) \le \min_{s_1' \in \mathcal{S}_1} I(X_2; Y|X_1, Q, s_1', s_2) \quad (5)$$

$$R_1(s_1) + R_2(s_2) \le I(X_1, X_2; Y|Q, s_1, s_2), \quad (6)$$

for all (s_1, s_2) and a set of joint pmfs of the form $p(q)p(x_1|s_1, q)p(x_2|s_2, q)$, $(s_1, s_2) \in \mathcal{S}_1 \times \mathcal{S}_2$.

Proof: We first prove the achievability of any rate tuple in the region.

Random code generation: Fix a set of joint pmfs $p(q)p(x_1|q, s_1)p(x_2|q, s_2)$, $(s_1, s_2) \in \mathcal{S}_1 \times \mathcal{S}_2$. Randomly generate a sequence q^n according to $\prod_{t=1}^n p(q_t)$. Conditioned on q^n and for each state $s_1 \in \mathcal{S}_1$ and $w_1 \in \mathcal{W}_1(s_1)$, generate $2^{nR_1(s_1)}$ conditionally independent $x_1^n(w_1|s_1)$ sequences according to $\prod_{t=1}^n p(x_{1t}|q_t, s_1)$. Similarly, generate $2^{nR_2(s_2)}$ conditionally independent $x_2^n(w_2|s_2)$ sequences according to $\prod_{t=1}^n p(x_{2t}|q_t, s_2)$ and $w_2 \in \mathcal{W}_2(s_2)$ for each $s_2 \in \mathcal{S}_2$.

Encoding: Each encoder observes its state and so to send w_1, Sender 1 sends $x_1^n(w_1|s_1)$, and to send w_2, Sender 2 sends $x_2^n(w_2|s_2)$.

Decoding: The decoder, knowing both s_1 and s_2, declares that $(\hat{w}_1, \hat{w}_2)$ is sent if it is the unique message pair such that $(q^n, x_1^n(\hat{w}_1|s_1), x_2^n(\hat{w}_2|s_2), y^n, s_1, s_2) \in A_\epsilon^{(n)}$, otherwise it declares an error. Here, $A_\epsilon^{(n)}$ denotes the set of weakly typical sequences [10].

The average probability of error for each state component pair can be bounded as for the DM-MAC (e.g., see [10]) to show that $P_e^{(n)}(s_1, s_2) \to 0$ as $n \to \infty$, provided that

$$R_1(s_1) \le I(X_1; Y|X_2, s_1, s_2) \quad (7)$$

$$R_2(s_2) \le I(X_2; Y|X_1, s_1, s_2)$$

$$R_1(s_1) + R_2(s_2) \le I(X_1, X_2; Y|s_1, s_2). \quad (8)$$

Since $P_e^{(n)}(s_1, s_2) \to 0$ for all possible pairs of $(s_1, s_2) \in \mathcal{S}_1 \times \mathcal{S}_2$ with codewords and rates that depend only on the distributed state information, the individual rates are upper-bounded by

$$R_1(s_1) \le \min_{s_2' \in \mathcal{S}_2} I(X_1; Y|X_2, s_1, s_2') \quad (9)$$

$$R_2(s_2) \le \min_{s_1' \in \mathcal{S}_1} I(X_2; Y|X_1, s_1', s_2). \quad (10)$$

However, for the sum-rate, $P_e^{(n)}(s_1, s_2) \to 0$ as long as (8) is satisfied and therefore no minimization is needed. This establishes the achievability of the rate region defined in (4)-(6).

Proof of the converse requires showing that given any set of $(2^{nR_1(1)}, \cdots, 2^{nR_1(S)}, 2^{nR_2(1)}, \cdots, 2^{nR_2(S)}, n)$ codes with $P_e^{(n)}(s_1, s_2) \to 0$, $(R_1(1), \cdots, R_1(S), R_2(1), \cdots, R_2(S))$ is in the region defined in (4)-(6). By Fano's inequality,

$$H(W_1(s_1)|Y^n, s_1, s_2) \le n\epsilon_n$$

$$H(W_2(s_2)|Y^n, s_1, s_2) \le n\epsilon_n,$$

where $\epsilon_n \to 0$ as $n \to \infty$, for all $(s_1, s_2) \in \mathcal{S}_1 \times \mathcal{S}_2$. The above inequalities imply that

$$H(W_1(s_1), W_2(s_2)|Y^n, s_1, s_2) \le 2n\epsilon_n$$

for any $(s_1, s_2) \in \mathcal{S}_1 \times \mathcal{S}_2$.

Using these inequalities and steps similar to those used in the converse proof for the DM-MAC (e.g. in [10]), we obtain

$$R_1(s_1) \le \frac{1}{n}\sum_{t=1}^n I(X_{1t}; Y_t|X_{2t}, s_1, s_2) + \epsilon_n \quad (11)$$

$$R_2(s_2) \le \frac{1}{n}\sum_{t=1}^n I(X_{2t}; Y_t|X_{1t}, s_1, s_2) + \epsilon_n \quad (12)$$

$$R_1(s_1) + R_2(s_2) \le \frac{1}{n}\sum_{t=1}^n I(X_{1t}, X_{2t}; Y_t|s_1, s_2) + \epsilon_n, \quad (13)$$

for all pairs $(s_1, s_2) \in \mathcal{S}_1 \times \mathcal{S}_2$. If we let Q be uniformly distributed over $1, \cdots, n$ and independent of X_1^n, X_2^n, Y^n, and define $X_1 = X_{1Q}$, $X_2 = X_{2Q}$, and $Y = Y_Q$, then,

$$\begin{aligned} R_1(s_1) &\le \frac{1}{n}\sum_{t=1}^n I(X_{1t}; Y_t|X_{2t}, s_1, s_2) + \epsilon_n & (14)\\ &= \frac{1}{n}\sum_{t=1}^n I(X_{1t}; Y_t|X_{2t}, s_1, s_2, Q=t) + \epsilon_n & (15)\\ &= I(X_{1Q}; Y_Q|X_{2Q}, s_1, s_2, Q) + \epsilon_n & (16)\\ &= I(X_1; Y|X_2, s_1, s_2, Q). & (17) \end{aligned}$$

Therefore, for every state component pair (s_1, s_2), the following inequalities must be satisfied:

$$R_1(s_1) \le I(X_1; Y|X_2, s_1, s_2, Q) \quad (18)$$

$$R_2(s_2) \le I(X_2; Y|X_1, s_1, s_2, Q)$$

$$R_1(s_1) + R_2(s_2) \le I(X_1, X_2; Y|s_1, s_2, Q),$$

for some joint pmf $p(q)p(x_1|s_1, q)p(x_2|s_2, q)$. Since these three inequalities should be satisfied for all state pairs $(s_1, s_2) \in \mathcal{S}_1 \times \mathcal{S}_2$, the rate of Sender 1, $R_1(s_1)$, must satisfy all the inequalities in (18) with different $s_2 \in \mathcal{S}_2$, and so we have

$$R_1(s_1) \le \min_{s_2' \in \mathcal{S}_2} I(X_1; Y|X_2, s_1, s_2', Q).$$

Similarly, the rate of sender 2, $R_2(s_2)$ must satisfy

$$R_2(s_2) \le \min_{s_1' \in \mathcal{S}_1} I(X_2; Y|X_1, s_1', s_2, Q).$$

This completes the proof of the converse. ■

Note that unlike the ergodic MAC capacity region, which can be described by the union of sets that are defined by 3 inequalities, the adaptive-rate capacity region is defined by $2S + S^2$ inequalities. For example, the rate of Sender 1 for $S_1 = 1$, i.e., $R_1(1)$, is constrained by S sum-rate inequalities in (6) as well as the individual rate inequality in (4). Moreover, the sum-rate constraint in (6) is not necessarily tight for each (s_1, s_2) pair, because for any fixed state component, the inequality must be satisfied for all possible states of the other state component.

In the i.i.d. state model, the state pair is assumed to be fixed during each block of n transmissions, where n is large enough to achieve low probability of decoding error. Assuming that the state pair (S_1, S_2) is drawn i.i.d. according to the probability mass function pmf $p(s_1, s_2)$, the expected sum-rate capacity of the DM-MAC-DSI is defined as

$$C_{\text{sum,dsi}} = \max_{p(q)p(x_1|s_1,q)p(x_2|s_2,q)} \mathbb{E}_{S_1,S_2}\left(R_1(S_1) + R_2(S_2)\right). \tag{19}$$

To obtain this sum-rate, one needs to consider not only the sum-rate inequality in (6) but also the individual-rate inequalities in (4) and (5), because this sum-rate inequality may not be tight.

We shall compare this expected adaptive sum-rate capacity to the ergodic sum-rate capacities when the senders know both state components [1] and when each sender knows only its state. Recall that the ergodic capacity region for the first case, when the state consists of two components, is the set of all rate pairs (R_1, R_2) such that

$$R_1 \le I(X_1; Y|X_2, S_1, S_2, Q) \tag{20}$$
$$R_2 \le I(X_2; Y|X_1, S_1, S_2, Q) \tag{21}$$
$$R_1 + R_2 \le I(X_1, X_2; Y|S_1, S_2, Q), \tag{22}$$

for some joint pmf of the form $p(q)p(x_1|s_1, s_2, q)$ $p(x_2|s_2, s_1, q)$.

The ergodic capacity region when each sender knows only its state has a similar form except that the allowed joint pmf is of the form $p(q)p(x_1|s_1, q)p(x_2|s_2, q)$. This follows from Theorem 5 in [7] by setting $S_{T1} = S_1, S_{T2} = S_2$, and $S_R = S_1, S_2$.

A. Binary DM-MAC with Distributed State Information

Consider the binary DM-MAC with two independent state components in Fig. 3. Each state component is distributed as a $\text{Bern}(1/2)^2$. The states 0 and 1, respectively, may indicate the absence and presence of a packet to transmit at each sender in a random access channel. This channel can be also viewed as a Gilbert channel where the state is either good or bad.

The adaptive-rate capacity region in Theorem 1 is achieved when we set $X_1, X_2 \sim \text{Bern}(1/2)$ and reduces to the set of all rate quadruples $(R_1(0), R_1(1), R_2(0), R_2(1))$ such that

$$\begin{gathered} R_1(0) \le 0,\ R_2(0) \le 0,\ R_1(1) \le 1, R_2(1) \le 1 \\ R_1(0) + R_2(0) \le 0,\ R_1(0) + R_2(1) \le 1, R_1(1) + R_2(0) \le 1 \\ R_1(1) + R_2(1) \le 1. \end{gathered} \tag{23}$$

Since $R_1(0) = R_2(0) = 0$ and $R_1(1) = 1 - R_2(1)$ from (23), the expected adaptive sum-rate capacity is given by

$$\frac{R_1(0) + R_2(1)}{4} + \frac{R_1(1) + R_2(0)}{4} + \frac{R_1(1) + R_2(1)}{4}$$
$$= \frac{1}{2}(R_1(1) + R_2(1)) = 0.5.$$

[2] $\text{Bern}(p)$ refers to a Bernoulli random variable with parameter p.

If we were to treat this channel as a compound channel, each sender would need to design a code for the worst state pair, which in this example corresponds to the $(0, 0)$ state. Note that in this state pair, the sum-rate is equal to zero. Therefore, allowing rate adaptation can greatly increase the expected sum-rate. It is easy to see that the ergodic sum-rate capacity for this channel is 0.75 for both the case when the senders know both states and when each sender knows only its own state. The expected adaptive sum-rate capacity is smaller because it is defined under a more stringent requirement on the probability of error. If coding over multiple blocks is allowed, a random inter-leaver can transform this i.i.d. state channel into an ergodic channel. This increases the expected sum-rate capacity to 0.75 but at the expense of much longer coding delay.

IV. Gaussian MAC with Distributed State Information

The adaptive-rate capacity region can be easily established for the Gaussian case as it can be readily shown that Gaussian signaling is optimal. The following proposition states the expected adaptive sum-rate capacity.

Proposition 1: The expected adaptive sum-rate capacity can be obtained by solving the convex optimization problem

$$\max \sum_{(h_1,h_2)} p(h_1, h_2)\left[R_1(h_1) + R_2(h_2)\right] \tag{24}$$

$$\text{s.t.} \sum_{h_k} p(h_k)P_k(h_k) \le P, \quad k = 1, 2$$

$$R_1(h_1) \le \frac{1}{2}\log_2\left(1 + \frac{|h_1|^2 P_1(h_1)}{N}\right) \quad \forall h_1 \tag{25}$$

$$R_2(h_2) \le \frac{1}{2}\log_2\left(1 + \frac{|h_2|^2 P_2(h_2)}{N}\right) \quad \forall h_2 \tag{26}$$

$$R_1(h_1) + R_2(h_2) \le \tag{27}$$
$$\frac{1}{2}\log_2\left(1 + \frac{|h_1|^2 P_1(h_1) + |h_2|^2 P_2(h_2)}{N}\right) \quad \forall h_1, h_2$$

$$P_1(h_1) \ge 0, \quad \forall h_1$$
$$P_2(h_2) \ge 0, \quad \forall h_2$$

where P is the long-term power constraint, $p(h_1, h_2)$ is the joint pmf of the fading coefficients, and $P_k(h_k)$ is the power allocated by Sender k to state realization h_k, $k = 1, 2$.

Unlike the general DM-MAC-DSI, here the other sender's effect can be completely eliminated at the decoder by knowing the other sender's transmit signal and the states; therefore, the inequalities in (25) and (26) require no minimization (unlike the inequalities (4) and (5)). Note that the sum-rate inequality (27) must be satisfied for all fading state pairs (h_1, h_2) and so the inequality is not tight in general.

We compare this adaptive sum-rate capacity to the ergodic sum-rate capacities for complete and distributed knowledge of the state at the senders. When the senders know both state components, the ergodic sum-rate capacity is obtained

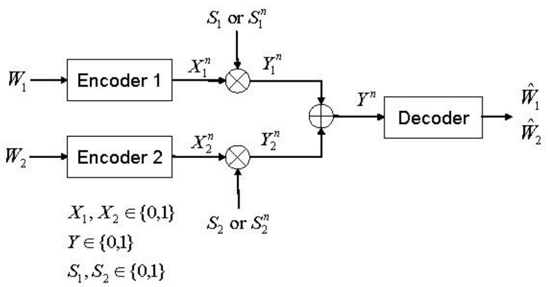

Fig. 3. Binary DM-MAC-DSI with independent states.

by solving the following convex optimization problem [1]:

$$\max \sum_{(h_1,h_2)} p(h_1,h_2)\frac{1}{2}\log_2\left[1+\frac{\sum_{k=1}^{2}|h_k|^2 P_k(h_1,h_2)}{N}\right]$$

$$\text{s.t.} \sum_{(h_1,h_2)} p(h_1,h_2)P_k(h_1,h_2) \le P, \quad k=1,2$$

$$P_k(h_1,h_2) \ge 0, \quad \forall (h_1,h_2), \quad k=1,2,$$

where P is the power constraint and $P_k(h_1,h_2)$ denotes the power allocated by Sender k for state (h_1,h_2). When each sender knows only its state, the ergodic sum-rate capacity has the same form except that the power of each sender can depend only on its state.

Fig. 4 compares the expected adaptive sum-rate capacity for the AWGN case to the ergodic sum-rate capacity and the sum-rate capacity under the compound channel setting, where codebooks are designed to support the least-capable state pair. We assume that the noise power is $N=1$, the average power constraint is $P=1$, and the fading coefficients H_1, H_2 are uniformly distributed over $\{1,6\}^2$. Thus, the signal to noise ratio (SNR) varies between -10 and 25dB. For a given SNR, the noise power becomes $(1+6^2)/2SNR$. The solutions to the above convex optimization problems were obtained using the CVX [3] and MOSEK [4] optimization packages.

As before, if we relax the constraint on the error probability per block, then we can achieve the ergodic sum-rate capacity using inter-leaving. The figure shows that the expected adaptive sum-rate capacity is close to the ergodic capacity with distributed state information. Therefore, there is no great loss in the sum-rate capacity for two senders. However, this loss increases as the number of senders increases. The compound channel capacity in this case is equivalent to the rate supported in the worst channel state, which is $(h_1,h_2) = (1,1)$. Therefore, without allowing the power allocation over blocks, the compound sum-rate capacity is $\frac{1}{2}\log_2\left(1+2SNR/(1+6^2)\right)$, which is much smaller than the sum-rate obtained from Proposition 1.

V. Conclusion

We formulated the problem of a multiple-access channel with distributed state information, where the receiver knows the entire state but each sender knows only a component of it. We established the adaptive-rate capacity region under the i.i.d state model, and showed that this adaptation can achieve a significantly higher sum-rate capacity than the more pessimistic compound channel approach.

Several interesting questions remain. For example, we only considered discrete fading states, so it would be interesting to extend the results to a continuous fading model such as Rayleigh fading. Further, the fading Gaussian MAC with distributed-state information requires further attention because of its many special characteristics and its practical importance. For example, the random access channel model can be combined with the Gaussian channel to accurately model wireless random access networks such as IEEE 802.11.

[3]Matlab Software for Disciplined Convex Programming, http://www.stanford.edu/ boyd/cvx, October 2006.

[4]Mosek ApS Optimization Software. http://www.mosek.com, October 2006.

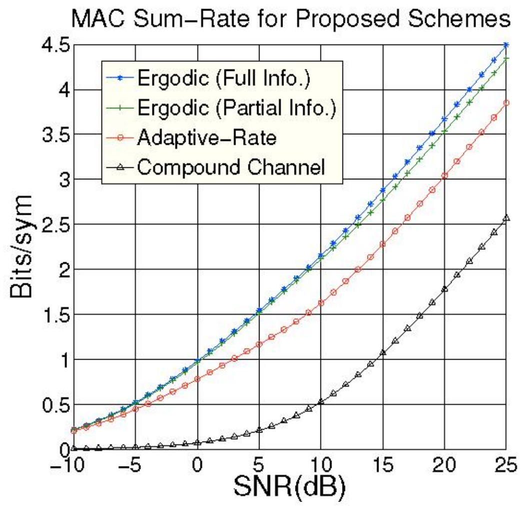

Fig. 4. Sum-rates in a Gaussian MAC with discrete states.

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